Optimization in Computer Vision

In the context of 3D scene reconstruction

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Ciência 2010
July 6, 2010

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Introduction

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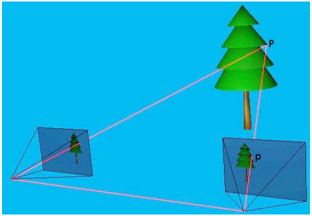
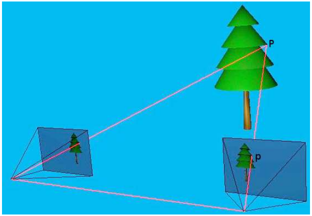
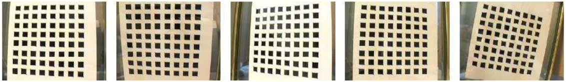
Camera Calibration



Camera Calibration





$$
\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{n} \sum_{j=1}^{n} \left\| \mathbf{p}_{ij} - f(\mathbf{A}_i, \mathbf{R}_i, \mathbf{t}_i, \mathbf{P}_j) \right\|^2 \\
\text{s.t.} \quad & \mathbf{A}_i \in \text{Intrinsic Parameters} \\
& \mathbf{R}_i \in \text{Camera Rotation} \\
& \mathbf{t}_i \in \text{Camera Translation}
\end{aligned}
$$

Zhang, Zhengyou. “Flexible Camera Calibration By Viewing a Plane From Unknown Orientations”. ICCV 1999

*Camera Calibration Toolbox*for Matlab by Jean-Yves Bouguet

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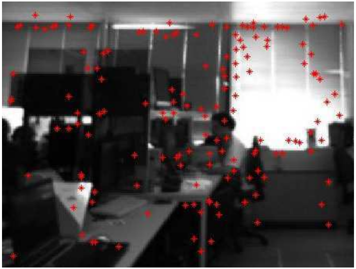
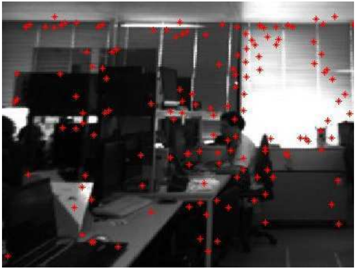
Sparse Matching - LSAP





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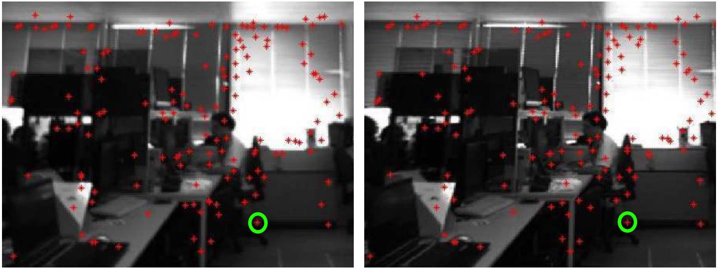
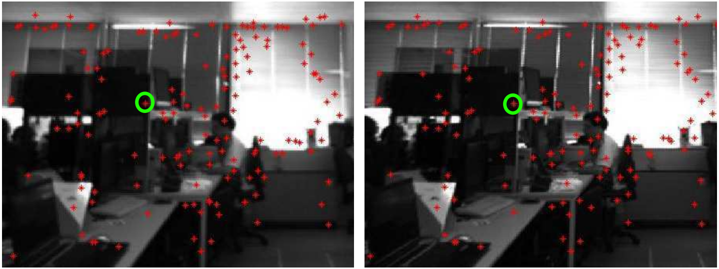
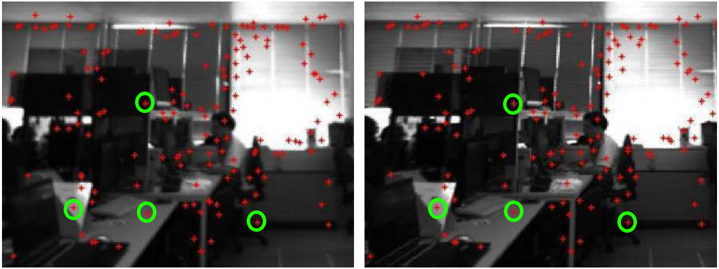
Sparse Matching - LSAP



Sparse Matching - LSAP



Sparse Matching - LSAP



$$\begin{bmatrix} .1 & .7 & .9 & .1 \\ .3 & .2 & .8 & .3 \\ .6 & .1 & .7 & .4 \\ .2 & .4 & .6 & .8 \end{bmatrix}$$

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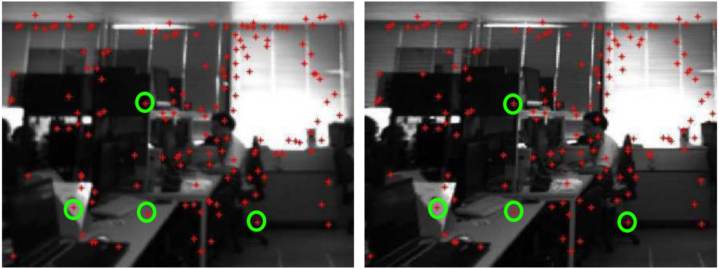
Sparse Matching - LSAP



$$\begin{bmatrix} .1 & .7 & .9 & .1 \\ .3 & .2 & .8 & .3 \\ .6 & .1 & .7 & .4 \\ .2 & .4 & .6 & .8 \end{bmatrix}$$

$$\begin{aligned} \text{maximize} \quad & \textstyle\sum_{ij} c_{ij} p_{ij} \\ \text{s.t.} \quad & p_{ij} \in \text{Permutation} \end{aligned}$$

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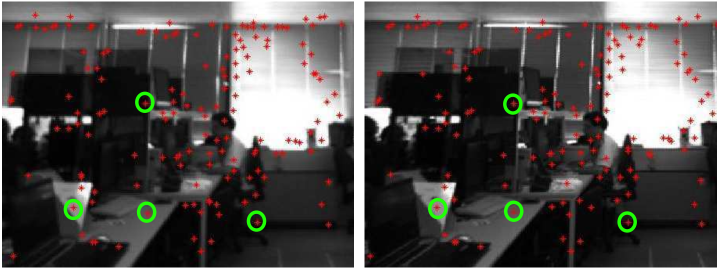
Sparse Matching - LSAP



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Value = 3.2

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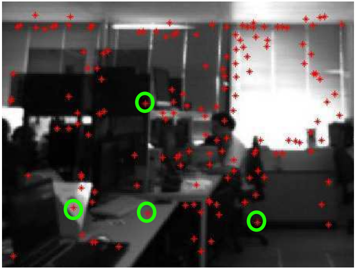
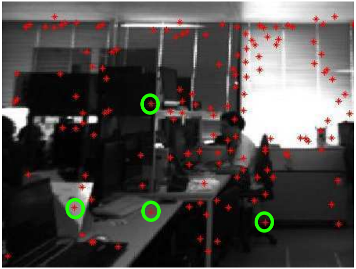
Sparse Matching - LSAP

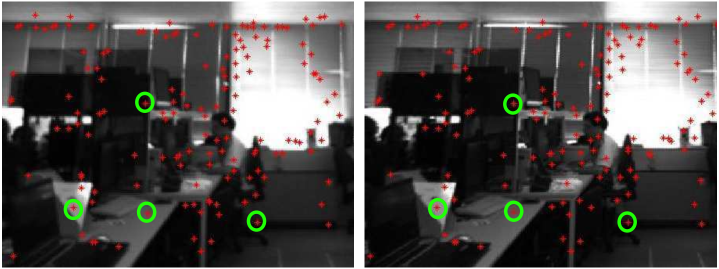



$$\begin{bmatrix} .1 & .7 & .9 & .1 \\ .3 & .2 & .8 & .3 \\ .6 & .1 & .7 & .4 \\ .2 & .4 & .6 & .8 \end{bmatrix}$$

$$\begin{aligned} \text{maximize} \quad & \textstyle\sum_{ij} c_{ij} p_{ij} \\ \text{s.t.} \quad & p_{ij} \in \text{Permutation} \end{aligned}$$

Value = 3.0

Sparse Matching - LSAP

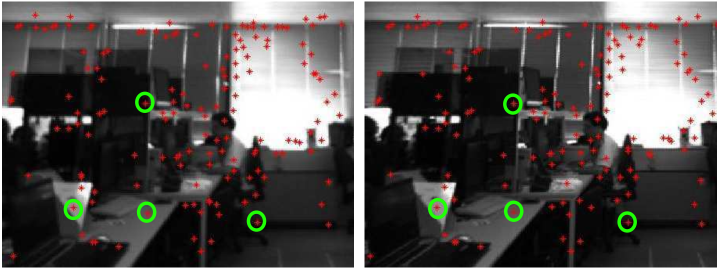


$$\begin{bmatrix} .1 & .7 & .9 & .1 \\ .3 & .2 & .8 & .3 \\ .6 & .1 & .7 & .4 \\ .2 & .4 & .6 & .8 \end{bmatrix}$$

$$\begin{array}{rl} \text{maximize} & \sum_{ij} c_{ij} p_{ij} \\ \text{s.t.} & p_{ij} \in \text{Permutation} \end{array}$$

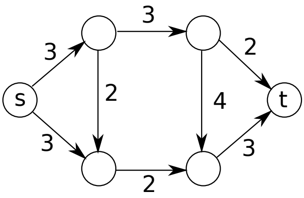
Value = 2.9

Sparse Matching - LSAP



- **Hungarian Algorithm** (Dénes Kõnig, Jenõ Egerváry)
- Harold Kuhn - James Munkres (strongly polynomial $O(n^4)$)
- Jack Edmonds - Richard Karp ($O(n^3)$)

Dense Matching - Maximum Flow



- What is the maximum flow possible from node “s” to node “t”?
- Solution is polynomial in time ($O(V^3)$ or $O(V\ E\ \log(V))$, or ...)

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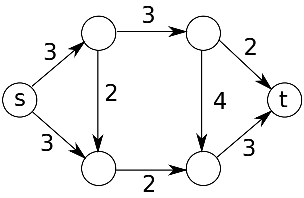
Dense Matching - Maximum Flow



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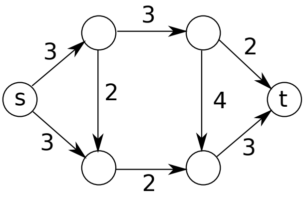
Dense Matching - Maximum Flow



- What is the maximum flow possible from node “s” to node “t”?
- Solution is polynomial in time ($O(V^3)$ or $O(V\ E\ \log(V))$, or ...)

$$\begin{aligned} \text{minimize} \quad & \mathbf{x}^T \mathbf{A} \mathbf{x} \\ \text{s.t.} \quad & \mathbf{x} \in \{0,1\}^n \end{aligned}$$

- In general is NP-hard.
- If outside the diagonal of **A** all elements are non-positive, can be solved using maximum flow.

Kolmogorov, Vladimir and Ramin Zabih. “What Energy Functions Can Be Minimized Via Graph-Cuts”. IEEE TPAMI. 2004.

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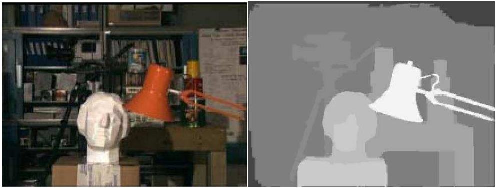
Dense Matching - Maximum Flow

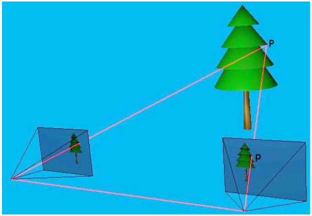
The multi-camera matching problem can be described as an optimization problem of the previous form



These are considered some of the best multi-camera reconstruction algorithms to date.

Kolmogorov, Vladimir and Ramin Zabih. "Multi-camera Scene Reconstruction via Graph Cuts". ECCV 2002.

Reconstruction



Epipolar constraint is not satisfied in the presence of noisy measurements.

$$\mathbf{p}_l^T \ \mathbf{F} \ \mathbf{p}_r = 0$$

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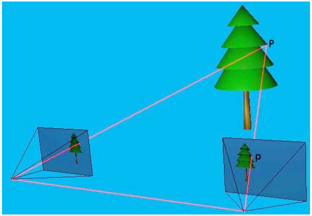
Reconstruction



Epipolar constraint is not satisfied in the presence of noisy measurements.

$$\mathbf{p}_l^T \ \mathbf{F} \ \mathbf{p}_r = 0$$

$$\begin{aligned} \text{minimize} \quad & d(\mathbf{x}_l, \mathbf{p}_l)^2 + d(\mathbf{x}_r, \mathbf{p}_r)^2 \\ \text{s.t.} \quad & \mathbf{x}_l^T \ \mathbf{F} \ \mathbf{x}_r = 0 \\ & \mathbf{x}_l \in \mathbb{R}^2 \\ & \mathbf{x}_r \in \mathbb{R}^2 \end{aligned}$$

Exact solution (roots of degree 6 polynomial)

Hartley, R. and Andrew Zisserman. “Multiple View Geometry in Computer Vision”. Cambridge University Press, 2000.

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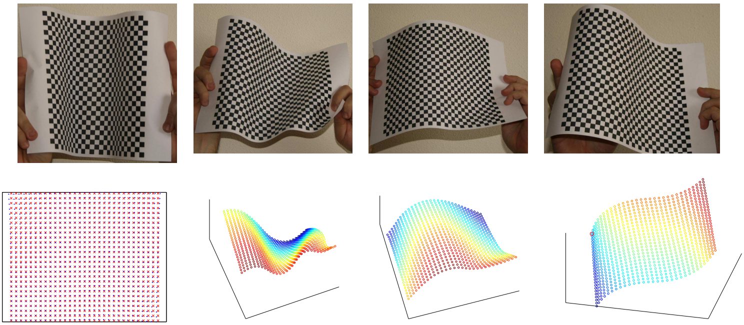
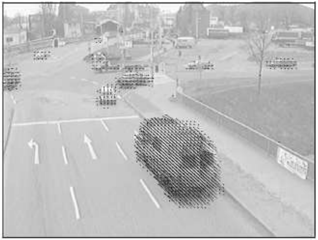
My Work - Non-Rigid (Isometric) Reconstructions



7 different optimization problems.

Other Problems in Computer Vision

- Optical Flow
- Image Segmentation
- Image Registration
- Simultaneous Localization and Mapping (*SLAM*)
- ...



Nils Papenberg, Andres Bruhn, Thomas Brox, Stephan Didas and Koachim Weickert. "Highly Accurate Optic Flow Computation with Theoretically Justified Warping", 2005.

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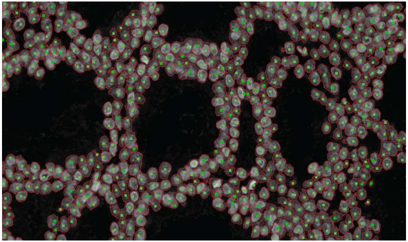

Al-Kofahi Y, Lassoued W, Lee W, Roysam B. “Improved Automatic Detection and Segmentation of Cell Nuclei in Histopathology Images”. IEEE Trans Biomed Eng 2009

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Hugin Tutorial

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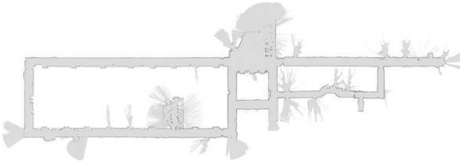

DP-SLAM Homepage

Conclusion

- (Almost) All problems in Computer Vision are naturally described as an optimization problem.

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END

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